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Vibration of structures with variable stiffness

E. Demirkan & N. Kadioglu
Faculty of Civil Engineering, Istanbul Technical University, Istanbul, Turkey

ABSTRACT: The general assumption is that stiffness matrix is constant in structures. But if the loads increase, deformation-load curve becomes nonlinear. The aim of this study is to analyze the vibration of a structure under these facts. The chosen sample problem is a steel single storey frame with a horizontal force. At first, classical stiffness matrix is determined under static loads. Then the structure has been reduced to a unique mass-spring system with increasing loads. It is accepted that material is ideal elasto-plastic. The change of stiffness under increasing loads will be calculated by plastic analysis. The system is not like the beginning after first plastic hinge occurs and degree of freedom of the system changes. If the load increases the decreasing of stiffness matrix continues. It is accepted that stiffness matrix is constant until first plastic hinge occurs. Also, it is accepted that the stiffness matrix is also constant between first and the second plastic hinges. By this way, a curve is obtained between load and stiffness matrix and the stiffness-force diagram has been plotted. In addition, the positions of plastic hinges have been controlled by SAP2000 computer program. Finally, the forced vibration of the sample structure under an harmonic load has been investigated. It is clear that the external load varies by time. Then the force-time diagram and the stiffness-time graphic have also been plotted under increasing loads. The damping coefficient of the system must be calculated for every stiffness value by choosing a initial value for system damping. The displacement-time curve has also been given. The operations of increasing loads have been repeated for decreasing loads. Also the displacement-time curves for increasing and decreasing loads have been combined and they have been shown on a single diagram.

1 INTRODUCTION

The aim of this study is to calculate the variable stiffness matrix under increasing and decreasing loads. As the first step the solution of any structure under static loads is solved by classical matrix-displacement method assuming stiffness matrix is constant. And the stress resultants and stresses are calculated in the bars forming the structure. Then the loads have been increased. As a result of this increment a specific cross section of a certain bar cannot carry more moment. Then a plastic hinge occur at this point. The new system is different from the original system. Again load is increased and new system is solved till a second plastic hinge occurs. Here only plane systems are considered and the moment at a plastic hinge is bending moment.

Degree of freedom of the first system decreases one degree after each plastic hinge occurs. Then a specific point is selected and the deformation of this point is calculated for every stiffness matrix. These procedure will continue till the system is unstable. And the load-deformation curve is plotted for this specific point and system is reduced a mass-spring system. And using deformation-load curve the variation of the stiffness of this imaginary spring by load is obtained.

After these same structure is examined under dynamic loads which vary by time. Dynamic solution is made using certain models. The main factors that determine results, the distribution of the masses in the structure, the characteristics of external forces and displacements, internal friction, cracking, the resistances against deformation of the bars(elements) and damping mechanism. (Yerlici and Lus, 2007).

Consider a system which can only rotate about an axis or can move in just one direction. This model is called a single degree of freedom system. The dynamic behavior of a structure under external influences that varies depending on the mass of system, the stiffness and energy loss in the system.

To solve a system under dynamic loads the equation of motion for the system is necessary. One equation is sufficient to determine the dynamic behavior of a single degree of freedom system.

2 SAMPLE PROBLEM

The chosen system is a plane frame with three bars which is shown in the Figure 1. A_1 and A_3 ends are connected to outer medium by built in connections. The chosen material is steel with young modulus 2.1×10^7 N/cm^2 and density 7850 kg/m^3 and yield stress is 275 N/mm^2. The material is assumed as ideal elasto-plastic. Degree of freedom is three.

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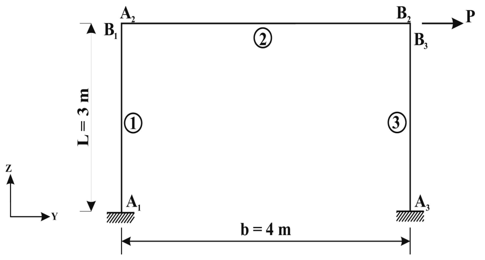

Figure 1. Sample problem.

Cross sections are square 6 × 6 cm for beam and 8 × 8 cm for columns.

There is only a horizontal P_1 force acting at point B_2. Horizontal displacement U_{B2} at point B_2 is determined by solving this system via matrix-displacement method. Here only this displacement is considered then the stiffness of the system is defined as

$$k_1 = \frac{P_1}{(U_{B2})_1} \tag{1}$$

Then the P load is increased and at one P_2 value first plastic hinge occurs. It is assumed that stiffness is constant till this value of P_2. First hinge is at A_3 point.

This system is also solved and new stiffness k_2 becomes

$$k_2 = \frac{P_2}{(U_{B2})_2} \tag{2}$$

Then this procedure continues till last plastic two hinges occur at A_2 and B_2. At this time this value of P is named as limit load. The variation of k versus P is given in Figure 2.

Then the dynamic loading is considered for the spring-mass system given by Figure 3

For this system equation of motion is

$$m\ddot{x} + c\dot{x} + kx = P(t) \tag{3}$$

Here k is the stiffness, m is the mass, c is the damping coefficient of the system and the force is assumed as a harmonic load as follows

$$P(t) = P\cos(\Omega t - \varnothing) = P\cos(\frac{2\pi}{T_F}t - \varnothing) \tag{4}$$

Here Ω is the frequency, T_F is period and $\varnothing$ is the phase angle which does not make any effect on reactions of the system. Then this term is ignored. The natural vibration of the system is also ignored. The chosen T_F is 0.4 sec for this problem.

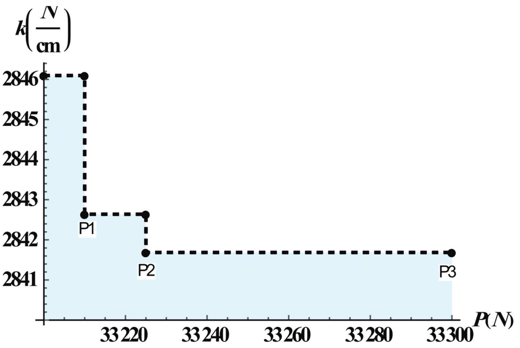

Figure 2. The variation of stiffness versus force.

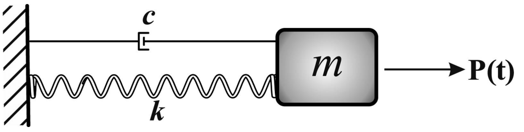

Figure 3. Spring-mass system.

Two new variables defined

$$\frac{c}{2m} = \xi\omega \tag{5}$$

$$\frac{k}{m} = \omega^2 \tag{6}$$

Here ξ is damping ratio and ω is natural frequency.

Three kinds of solutions can be found for equation (3) these are

- $\xi > 1$ heavy damping case
- $\xi = 1$ critical damping case
- $\xi < 1$ weak damping case

Here weak damping is considered and the solution of equation (3) becomes

$$\alpha_{1,2} = \omega\left(-\xi \mp i\sqrt{1-\xi^2}\right) \tag{7}$$

$$a = -\xi\omega \qquad b = \omega\sqrt{1-\xi^2} \tag{8}$$

$$x = e^{-at}\left(\left(\frac{1}{b}\int_0^t P(t)e^{at}\cos bt\,dt + C_1\right)\sin bt + \left(-\frac{1}{b}\int_0^t P(t)e^{at}\sin bt\,dt + C_2\right)\right)\cos bt \tag{9}$$

It is clear that the external load varies by time in the dynamic loading. Therefore, the time values have been evaluated for increasing load values using the load values at which the plastic hinges occur.

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Mass of the system is taken as

$$m = 115.23\ N.s^2 / m \tag{10}$$

Then the force-time diagram has been plotted under increasing loads in Figure 4.

After that, the stiffness-time graphic has also been given which shows time intervals of this stiffness.

Natural frequency values of the system have been calculated for every time interval.

$$\frac{k_1}{m} = \omega_1^2 \rightarrow \omega_1 = 49.698\ rad/\sec \tag{11}$$

$$\frac{k_2}{m} = \omega_2^2 \rightarrow \omega_2 = 49.668\ rad/\sec \tag{12}$$

$$\frac{k_3}{m} = \omega_3^2 \rightarrow \omega_3 = 49.659\ rad/\sec \tag{13}$$

The damping coefficient of the system must be calculated for every stiffness value by choosing a initial ξ_1 value for system damping.

$$\xi_1 = 0.02 \tag{14}$$

Table 1. The variation of load versus time.

	Load (*N*)	*Time*(sec)
P_1	33210	0.095
P_2	33225	0.096
P_3	33300	0.1

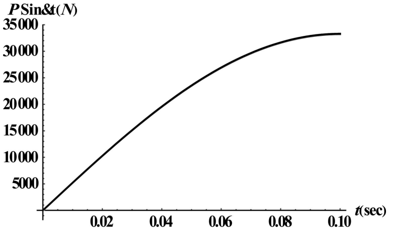

Figure 4. The variation of force versus time.

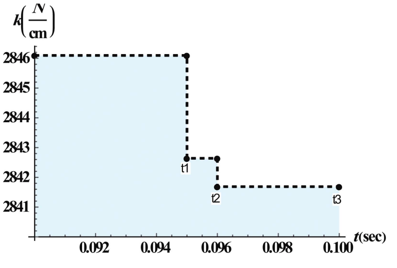

Figure 5. The variation of stiffness versus time.

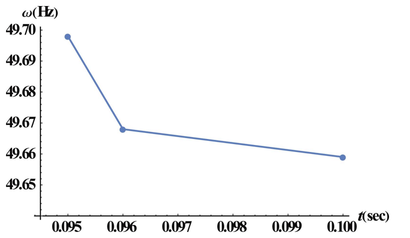

Figure 6. The variation of natural frequency versus time.

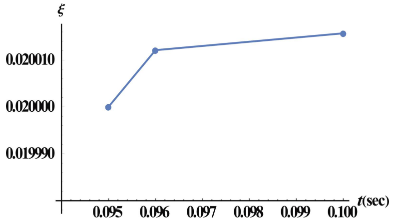

Figure 7. The variation of damping ratio versus time.

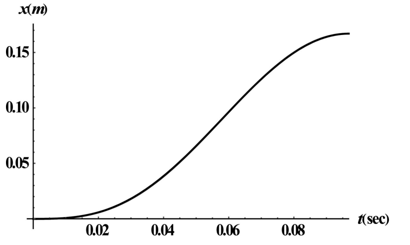

Figure 8. The variation of displacement versus time.

$$\frac{c}{m} = 2\xi_1\omega_1 \rightarrow c = 229.068\,N.s/m \tag{15}$$

$$\frac{c}{m} = 2\xi_2\omega_2 \rightarrow \xi_2 = 0.0200121 \tag{16}$$

$$\frac{c}{m} = 2\xi_3\omega_3 \rightarrow \xi_3 = 0.0200157 \tag{17}$$

According to this, the natural frequency-time and damping ratio of system-time graphics have been plotted.

It is assumed that the stiffness matrix remains constant between two values of the load when using the stiffness-force diagram. The system has been solved in a time interval that corresponds to these loads. The values, at the end of the previous interval have been used as initial conditions to determine the integration constants of the following interval. And the displacement-time curve has been plotted in Figure 8.

Table 2. The variation of load versus time.

	Load(N)	*Time*(sec)
P_1	33300	0.104
P_2	33225	0.105
P_3	33210	0.2

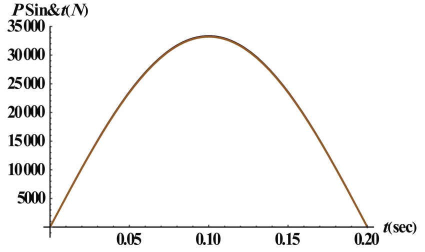

Figure 9. The variation of load versus time.

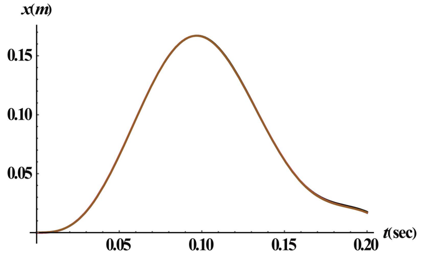

Figure 10. The variation of displacement versus time.

The operations of increasing loads have been repeated for decreasing loads. The changes of increased and decreased loads depending on time, has been represented in a single curve.

During calculations, the stiffness values of every time range that is calculated for increasing loads and decreasing loads have been taken as the same. The displacement-time curve, for decreasing loads has been plotted under these conditions. The displacement-time curves for increasing loads and decreasing loads have been combined and they have been shown on one diagram.

3 RESULTS

The variation of the stiffness matrix by increasing loads has been calculated. And this variation is used for dynamic loading. The selected problem has been reduced a single mass-spring system. But if a set of system deformations is considered, stiffness transforms to a matrix. Same procedure can be extended to all calculations. But a restriction is necessary. Whole exterior loads are horizontal and have the same time dependencies.

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